CSU FRESNO MATH PROBLEM SOLVING

February 28, 2009

Part 1: Counting & Probability

Counting: products and powers (multiply the number of choices idea)

1. (MH 11-12 2008) Suppose we draw 100 horizontal lines and 100 vertical lines in the plane. How many "pieces" of the plane are formed by cutting along all of these lines? Note that some of the pieces may have infinite area.

   (a) 10000

   (b) 10001

   (c) 10004

   (d) 10201

   (e) 10204

   Solution. When we cut a piece of paper along 100 parallel lines, we get 101 pieces. Here we are cutting in two directions, therefore the number of pieces is
   $101 \times 101 = (100+1)(100+1) = 100^2 + 2 \cdot 100 \cdot 1 + 1 = 10201$.

2. (MH 11-12 2008) How many subsets of $\{a, b, c, d, e, f, g\}$ contain both a and b?

   (a) 32

   (b) 25

   (c) 16

   (d) 12

   (e) 9

   Solution. A subset may or may not contain c, d, e, f, and g. So there are 2 choices for each of these letters (it is either an element of a subset or not). The total number of choices for 5 letters is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$.

3. (simplified MH 11-12 2008) The number of positive integers that are divisors of $720 = 2^4 \cdot 3^2 \cdot 5$ is

   (a) 16

   (b) 24

   (c) 25

   (d) 27

   (e) 30

   Solution. A positive integer n is a divisor of 720 if and only if its prime factorization contains at most four 2's, at most two 3's, at most one 5, and no other prime factors. So the exponent of 2 in the prime factorization of n has 5 choices (it can be 0, 1, 2, 3, or 4), the exponent of 3 has 3 choices (0, 1, or 2), and the exponent of 5 has 2 choices (0 or 1). The total number of choices is the product of these: $5 \cdot 3 \cdot 2 = 30$. Therefore the number 720 has 30 positive divisors.

4. (MH 11-12 2008) The number of **even** positive integers that are divisors of $720 = 2^4 \cdot 3^2 \cdot 5$ is
   (a) 15
   (b) 16
   (c) 24
   (d) 25
   (e) 29

   Solution. This problem is similar to the previous one, however, if n has to be even, then the factor 2 must be appear (at least once) in its prime factorization. So the exponent of 2 has only 4 choices: it can be 1, 2, 3, or 4. The rest is the same. Therefore there are $4 \cdot 3 \cdot 2 = 24$ even divisors.

Counting: factorials and binomial coefficients (the number of choices "keeps changing" idea)

5. (MH 9-10 2008) Ten people are attending a meeting. If each shakes hands with each other person exactly once, how many handshakes will occur?
   (a) 20
   (b) 45
   (c) 90
   (d) 100

   Solution. To count the number of pairs, we note that there are 10 ways to choose one person out of 10. Once the first person has been chosen, there are 9 ways to choose the second person. So we have $10 \cdot 9 = 90$ choices total. However, we counted each pair twice: each pair $\{A, B\}$ was counted as both $\{A, B\}$ and $\{B, A\}$. So we have to divide the number of choices by 2, and we get: $\frac{10 \cdot 9}{2} = 45$.
   More generally:
   The number of ways to choose k objects out of n objects is $\frac{n(n-1)(n-2) \cdot \ldots \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot 1}$, because there are n ways to choose the first object, $n-1$ ways to choose the second object after the first has been chosen, and so on. However, this way we count each k-tuple $k \cdot (k-1) \cdot (k-2) \cdot 1$ times because given k objects there are $k \cdot (k-1) \cdot (k-2) \cdot 1$ ways to order them (using the same idea of counting here). So we divide to account for multiple-counting.

   Notation and terminology. The product $1 \cdot 2 \cdot 3 \cdot \ldots \cdot k$ is called "k factorial" and is denoted $k!$.
   The number of ways to choose k objects out of n objects is called "n choose k" and is denoted $\binom{n}{k}$. So

$$\binom{n}{k} = \frac{n(n-1)(n-2) \cdot \ldots \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot 1} = \frac{n!}{k!(n-k)!}$$

   Example. In the above problem, we computed the number of ways to choose 2 people out of 10, i.e. $\binom{10}{2} = \frac{10!}{2!8!} = \frac{10 \cdot 9}{2} = 45$.

6. (MH 9-10 2008, MH 11-12 2008) At a party, every two people shook hands once. How many people attended the party if there were exactly 66 handshakes?
   (a) 65

(b) 54

(c) 33

(d) 22

(e) 12

Solution. Using the counting idea used in the previous problem, we have that for n people there would be $\frac{n(n-1)}{2}$ handshakes. So we have $\frac{n(n-1)}{2} = 66 = 6 \times 11$, then $n(n-1) = 12 \cdot 11$, so $n = 12$.

Binomial expansion

7. (MH 9-10 2008) Find the second term in the expansion of $(x+y)^{25}$.

(a) $2300x^{22}y^3$

(b) $300y^{223}x^2$

(c) $300x^{23}y^2$

(d) $25x^{24}y$

Solution. This problem assumes that we write the expansion in the "standard" way: first the term with the highest power of x, i.e. x^{25}, then the term with the next highest power of x, i.e. $Cx^{24}y$, and so on, until we reach the last term, without x: y^{25}. So the expression will have the form $x^{25} + C_1x^{24}y + C_2x^{23}y^2 + \ldots + C_{24}xy^{24} + y^{25}$.
The coefficients in this expression are the binomial coefficients: $C_i = \binom{25}{i}$, because each coefficient C_i is equal to the number of ways to choose i factors out of 25 (e.g. the i factors that will contribute a y to the product, and the other $25-i$ factors will contribute an x).
In particular, the coefficient $C_1 = \binom{25}{1} = 25$, so the second term is $25x^{24}y$.

Counting: other techniques

8. (MH 11-12 2008) In how many ways can you walk up a stairway with 6 steps if you can take one or two steps at a time?

(a) 9

(b) 10

(c) 11

(d) 12

(e) 13

Solution. First we can take either one or two steps. If we take one step, we'll have 5 steps left. If we take two steps, we'll have 4 steps left. So the number of ways to walk up a stairway with 6 steps is equal to the sum of the number of ways to walk up a stairway with 5 steps and the number of ways to walk up a stairway with 4 steps. The same idea can be used for a stairway with 5 steps, and 4 steps, and so on. If we denote the number of ways to walk up the stairway with n steps by S_n, then
$S_6 = S_5 + S_4 = (S_4 + S_3) + S_4 = 2S_4 + S_3 = 2(S_3 + S_2) + S_3 = 3S_3 + 2S_2 = 3(S_2 + S_1) + 2S_2 = 5S_2 + 3S_1$.
Clearly, $S_2 = 2$ and $S_1 = 1$, so $S_6 = 5 \cdot 2 + 3 \cdot 1 = 13$.

Probability: count the number of possible outcomes

9. (MH 9-10 2008) The land of Xod has coins that are regular triangular pyramids. The four faces are labeled N, G, H, and S. A Xodian is tossing two coins. What is the probability that both coins land with the same side facing down?

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{6}$

(d) $\frac{1}{8}$

Solution 1. Let's look at one the coins first and see which side is facind down. Now, what is the probability that the other coins has the same side facind down? Since there are 4 sides, the probability of the same side is $\dfrac{1}{4}$.

Solution 2. There are 16 possible outcomes total for two coins: NN, NG, NH, NS, GN, GG, GH, GS, HN, HG, HH, HS, SN, SG, SH, SS. Four of them have the same outcomes for both coins: NN, GG, HH, and SS. So the probability of this is $\dfrac{4}{16} = \dfrac{1}{4}$.

10. (MH 11-12 2008) What is the probability of rolling a red die and a blue die and having the number showing in the red die to be larger than the number showing in the blue one?

(a) $\frac{4}{9}$

(b) $\frac{1}{2}$

(c) $\frac{19}{36}$

(d) $\frac{2}{3}$

(e) $\frac{5}{12}$

Solution. First we should note that this problem assumes that the dice have the "standard" numbers on them: from 1 to 6. Since each die has 6 possible outcomes, the total number of outcomes is $6 \cdot 6 = 36$. Of these, 6 are when the two numbers are equal. So in 30 outcomes the two numbers are different. By symmetry, in 15 outcomes the number on the red die must be larger than the number on the blue die, and in 15 outcomes it will smaller. So the probability we need is $\dfrac{15}{36} = \dfrac{5}{12}$.

11. (MH 11-12 2008) A pair of dice is thrown. What is the probability that the two numbers that appear differ by exactly 2?

(a) $\frac{2}{3}$

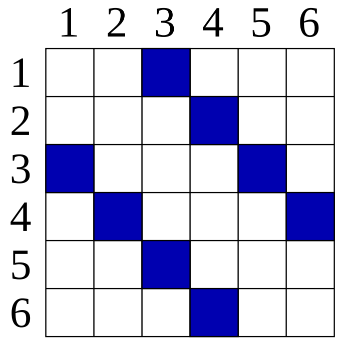
(b) $\frac{1}{3}$

(c) $\frac{1}{6}$

(d) $\frac{2}{9}$

(e) $\frac{1}{9}$

Solution. As noted in the previous problem, in 6 (out of 36 total) outcomes the two numbers are equal. In this problem we need to study the possible outcomes further. It is probably faster to make a table than try to list all 36 outcomes. Each cell in the table represents a possible outcome, and we shade the cells representing those outcomes in which the difference is exactly 2.



The probability of getting one of 8 of these outcomes out of 36 total is $\dfrac{8}{36} = \dfrac{2}{9}$.

Probability: earlier events affect the probability of the later events

12. (MH 9-10 2008) A jar contains 3 red marbles, 4 blue marbles, and 5 green marbles. Janice takes one marble out of the jar, and then Tom takes one marble out of the same jar. What is the probability that Janice drew a red marble and Tom drew a green marble?

    (a) $\frac{5}{44}$

    (b) $\frac{5}{48}$

    (c) $\frac{5}{33}$

    (d) $\frac{5}{36}$

    Solution. Since there are 12 marbles total and only 3 of them are red, the probability that Janice drew a red marble is $\frac{3}{12}$. If she drew a red marble, then only 2 red marbles, 4 blue marbles, and 5 green marbles (11 total) remained in the jar. The probability that Tom drew a green marble is $\frac{5}{11}$. The total probability of these two events is $\dfrac{3}{12} \cdot \dfrac{5}{11} = \dfrac{5}{44}$.

13. (MH 11-12 2008) An urn contains three white and four black balls. We take out a ball and put it in a drawer without looking at it. After that we take out a second ball. Find the probability that this ball is white.

    (a) $\frac{1}{6}$

    (b) $\frac{3}{7}$

    (c) $\frac{5}{6}$

    (d) $\frac{1}{3}$

    (e) $\frac{1}{7}$

    Solution. Since we do not know what the first ball was, we must consider both possibilities (with their probabilities). The probability that the first ball was white is $\frac{3}{7}$. Then only two white and four black balls remained in the urn, so the probability that the second ball is white is $\frac{2}{6}$. The total probability of this case (both balls white) is $\dfrac{3}{7} \cdot \dfrac{2}{6} = \dfrac{1}{7}$. Next, the probability that the first ball was black is $\frac{4}{7}$. Then three white and three black balls remained in the urn, so the probability that the second ball is white is $\frac{3}{6}$. The total probability of this case (first ball black, second white) is $\dfrac{4}{7} \cdot \dfrac{3}{6} = \dfrac{2}{7}$. Thus the total probability that the second ball is white is $\dfrac{1}{7} + \dfrac{2}{7} = \dfrac{3}{7}$.

14. (MH 9-10 2008) A color-blind individual has 16 pairs of socks, 10 identical red pairs and 6 identical navy blue pairs. After washing his socks, he just throws them in the sock drawer without pairing them up. If he randomly selects two socks, what is the probabiliy that they will be the same color?

    (a) $\frac{95}{248}$
    (b) $\frac{33}{248}$
    (c) $\frac{16}{31}$
    (d) $\frac{1}{2}$

    Solution. There are 32 socks total: 20 red and 12 navy blue. The probability that he selects two red is $\frac{20}{32}\cdot\frac{19}{31}$, and the probability that he selects two navy blue is $\frac{12}{32}\cdot\frac{11}{31}$. Thus the total probability that the two socks are the same color is $\dfrac{20}{32}\cdot\dfrac{19}{31}+\dfrac{12}{32}\cdot\dfrac{11}{31}=\dfrac{5}{8}\cdot\dfrac{19}{31}+\dfrac{3}{8}\cdot\dfrac{11}{31}=\dfrac{5\cdot 19+3\cdot 11}{8\cdot 31}=\dfrac{95+33}{8\cdot 31}=\dfrac{128}{8\cdot 31}=\dfrac{16}{31}$.

15. (MH 9-10 2008) A population starts with a single amoeba. For this one and for the generations thereafter, there is a probability of $\frac{3}{4}$ that an individual amoeba will split to create two amoebas, and a $\frac{1}{4}$ probability that it will die out without producing offsprings. What is the probability that the family tree of the original amoeba will go on for ever?

    (a) $\frac{1}{2}$
    (b) $\frac{1}{3}$
    (c) $\frac{2}{3}$
    (d) $\frac{3}{4}$

    Solution. This is one of the trickiest Math Field Day problems. Note that for all the amoebas of this kind, the probability that the family tree will go on for ever is the same. So let x be this probability. Then $1-x$ is the probability that it has a finite family tree. Another observation is that the family tree of the original amoeba will go on for ever if and only if it splits into two amoebas and at least one of the resulting amoebas has an infinite family tree. The probability that it will split is $\frac{3}{4}$ and the probability that at least one of the resulting amoebas has an infinite family tree is $1-(1-x)(1-x)$ because the probability that both of them will have finite familty trees is $(1-x)(1-x)$. So we have
    $x=\dfrac{3}{4}(1-(1-x)(1-x))$
    $4x=3(1-(1-2x+x^2))$
    $4x=3(1-1+2x-x^2)$
    $4x=3(2x-x^2)$
    $4x=6x-3x^2$
    $3x^2=2x$
    This equation has two roots: $x=0$ and $x=\frac{2}{3}$. Since 0 is not one of the choices, the answer must be $\frac{2}{3}$.

Part 2: Pythagorean Theorem, Area, and Volume

Pythagorean Theorem

1. (MH 9-10 2008) The hypotenuse c and one side b of a right triangle are consecutive integers. The square of the other leg of the triangle is

   (a) bc

   (b) $\frac{c}{b}$

   (c) $c+b$

   (d) $c-b$

   Solution. Using the Pythagorean theorem, the square of the other leg is $c^2-b^2=(c-b)(c+b)=c+b$ because $c-b=1$.

2. (MH 11-12 2008) Two perpendicular lines, intersecting at the center of a circle of radius 1, divide the circe into four parts. A smaller circle is inscribed in one of those parts. What is the radius of the smaller circle?

   (a) $\frac{1}{3}$

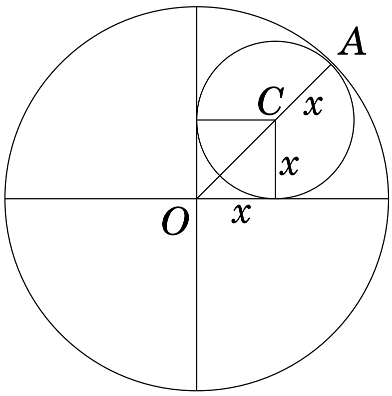
   (b) $\frac{2}{5}$

   (c) $\frac{1}{2}$

   (d) $\sqrt{2}-1$

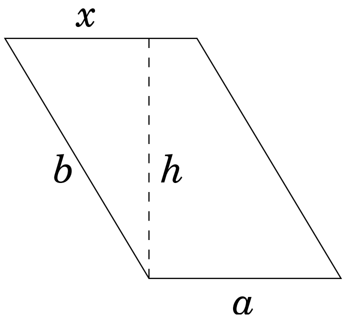
   (e) $2-\sqrt{2}$

   Solution. Let the radius of the smaller circle be x.



   Then, by Pythagorean theorem, the distance beween the centers of the two circles is $\sqrt{2}x$. Thus the radius of the large circle is $\sqrt{2}x+x=1$.
   So $(\sqrt{2}+1)x=1$,
   $$x=\frac{1}{\sqrt{2}+1}=\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1.$$

3. (MH 11-12 2008) Determine the length of the diagonals of the parallelogram shown if $a=6$ in, $b=10$ in and $h=8$ in.



(a) 10 in and 8 in

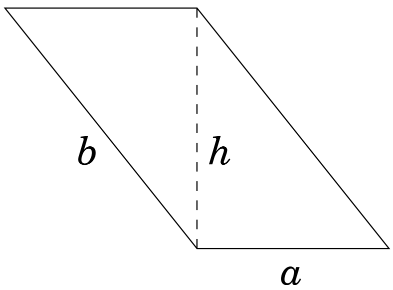
(b) 10 in and 6 in

(c) $2\sqrt{137}$ in and 8 in

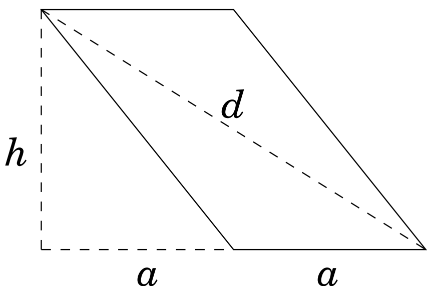
(d) $4\sqrt{13}$ in and 8 in

(e) $4\sqrt{13}$ in and $4\sqrt{13}$ in

<u>Solution.</u> First use the Pythagorean theorem to find the distance x (between a vertex and the base of the height): $x^2 + h^2 = b^2$, so $x^2 + 64 = 100$, thus $x^2 = 36$, and $x = 6$. We get that $x = a$, so the height is actually a diagonal – the picture was general, not quite accurate in this case! Here is a better picture:



To find the second diagonal, let's draw another height and extend one side so that to get a right triangle whose hypothenuse is that second diagonal:



Then by Pythagorean theorem again, $d^2 = (2a)^2 + h^2 = 12^2 + 8^2 = 144 + 64 = 208$, so $d = \sqrt{208} = 4\sqrt{13}$. Therefore the answer is $4\sqrt{13}$ in and 8 in.

4. (MH 9-10 2008) The ratio of the area of a square inscribed in a semicircle to the area of the square inscribed in the entire circle is

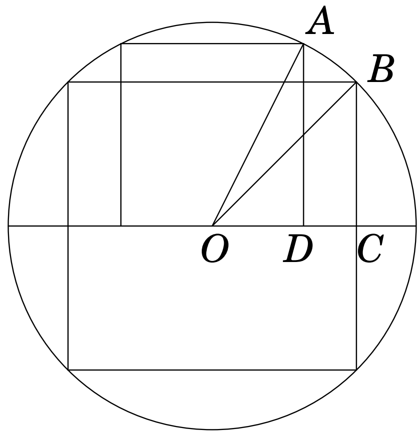
(a) 1:2

(b) 2:3

(c) 2:5

(d) 3:4

Solution. Let r be the radius of the circle, let x be the side of the square inscribed in the semicircle, and let y be the side of the square inscribed in the entire circle.



Then, using Pythagorean theorems for triangles OAD and OBC, we get:

$\left(\frac{x}{2}\right)^2 + x^2 = r^2$ and $\left(\frac{y}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = r^2$

$$\left(\frac{x}{2}\right)^2 + x^2 = \left(\frac{y}{2}\right)^2 + \left(\frac{y}{2}\right)^2$$

$$\frac{x^2}{4} + x^2 = \frac{y^2}{4} + \frac{y^2}{4}$$

$$\frac{5x^2}{4} = \frac{2y^2}{4}$$

$$5x^2 = 2y^2$$

$$\frac{x^2}{y^2} = \frac{2}{5}$$

So the ratio of the areas is 2 : 5.

5. (MH 11-12 2008) Two parallel chords in a circle have lengths 6 and 8. The distance between them is 1. Then the diameter of the circle is

   (a) $10\sqrt{3}$

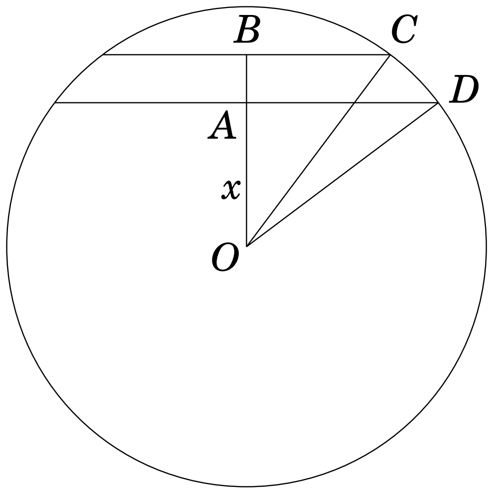
   (b) 14

   (c) 12

   (d) 10

   (e) 9

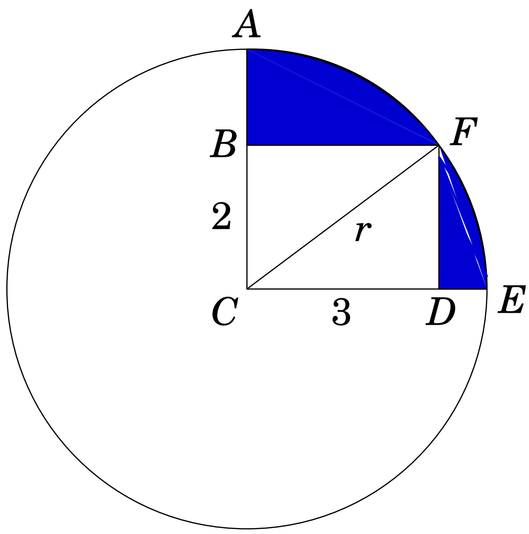
   Solution. Let $x = |OA|$ as shown in the figure below and let r be the radius of the circle. Also note that $|BC| = 3$, $|AD| = 4$, and $|AB| = 1$.



Using Pythagorean theorem for triangles OBC and OAD gives:
$(x+1)^2+3^2=r^2$, and
$x^2+4^2=r^2$.
Therefore $(x+1)^2+3^2=x^2+4^2$
$x^2+2x+1+9=x^2+16$
$2x+10=16$
$2x=6$
$x=3$.
Then $r^2=3^2+4^2=9+16=25$, so the radius is $r=5$ and the diameter is 10.

Pythagorean theorem and area

6. (MH 11-12 2008) C is the center of the circle and F is a point on the circle such that $BCDF$ is a 2 in by 3 in rectangle. What is the area of the shaded region (in square inches)?

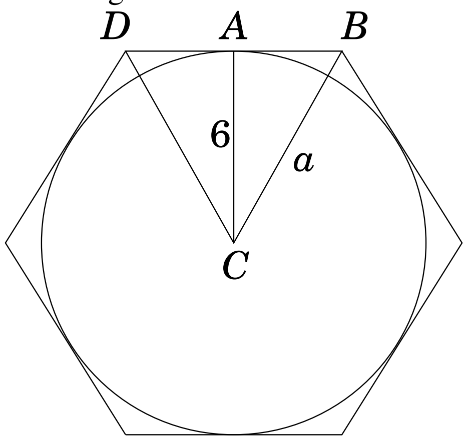


(a) $\frac{13\pi}{2}-5$
(b) $\frac{13\pi}{4}-5$
(c) $\frac{13\pi}{2}-6$
(d) $\frac{13\pi}{4}-6$

Solution. Using the Pythagorean theorem, we find that $r^2=2^2+3^2=13$. Therefore the area of one quarter of the circle is $\frac{\pi r^2}{4} = \frac{13\pi}{4}$. The area of the 2×3 rectangle is 6, so the area of the shaded region is $\dfrac{13\pi}{4} - 6$.

7. (MH 9-10 2008) A circle of radius 6 is inscribed in a regular hexagon. If the area of the hexagon is $x\sqrt{3}$, what is x?
   (a) 75
   (b) 72
   (c) 71
   (d) 70

   Solution. Let a be the side of the hexagon.



Since a regular hexagon can be divided into equilateral triangles (triangle BCD is one of them), $|BC| = a$, and $|AB| = \frac{a}{2}$, and the Pythagorean theorem for triangle ABC gives

$\left(\dfrac{a}{2}\right)^2 + 6^2 = a^2$

$\dfrac{a^2}{4} + 6^2 = a^2$

$\dfrac{3a^2}{4} = 36$

$\dfrac{a^2}{4} = 12$

$a^2 = 48$

$a = 4\sqrt{3}$

The area of triangle BCD is $\dfrac{1}{2}|BD| \cdot |AC| = \dfrac{1}{2} \cdot 4\sqrt{3} \cdot 6 = 12\sqrt{3}$, therefore the area of the whole hexagon is 6 times larger, i.e. $72\sqrt{3}$. Thus $x = 72$.

Various area problems

8. (MH 11-12 2008) Which of the following shapes has the largest area?
   (a) A right triangle with legs of length 6 and 8, and hypothenuse of length 10
   (b) A square with side of length 5
   (c) A circle with radius of length 3

(d) A rectangle with sides of length 3 and 9

Solution. Let's just find the areas of all these shapes. The area of the right triangle with legs of length 6 and 8 is $\frac{1}{2} \cdot 6 \cdot 8 = 24$. The area of the square with side of length 5 is $5^2 = 25$. The area of the circle with radius of length 3 is $\pi(3)^2 = 9\pi$. The area of the rectangle with sides of length 3 and 9 is 27. Since $\pi > 3$, $9\pi > 27$, so the circle has the largest area.

9. (MH 9-10 2008) If the length of each side of a triangle is increased by 20%, then the area is increased by what percent?

   (a) 40%

   (b) 44%

   (c) 48%

   (d) 52%

   Solution. The area is proportional to the square of the length. So if the length of each side is multiplied by 1.2 (which is equivalent to the increase by 20%), then the area will be multiplied by $1.2^2 = 1.44$, i.e. will increase by 44%.

10. (MH 11-12 2008) When the base of a triangle is decreased 10% and the altitude is increased 10%, then the area is

    (a) Unchanged

    (b) increased 10 %

    (c) decreased 10 %

    (d) increased 1 %

    (e) decreased 1 %

    Solution. Let b and h denote the base and height (altitude), respectively, of the original triangle. Its area is $A = \frac{bh}{2}$. The new base and height will be $1.1b$ and $0.9h$, and the new area is $A' = \frac{1.1b \cdot 0.9h}{2} = \frac{0.99bh}{2} = 0.99A$. So the area decreased by 1 %.

11. (MH 11-12 2008) If the width of a particular rectangle is doubled and the length is increased by three, then the area is tripled. What is the length of the rectangle?

    (a) 1

    (b) 2

    (c) 3

    (d) 6

    (e) 9

    Solution. Let w and l denote the width and length, respectively, of the original rectangle. Let $A = wl$ be its area. The width, length, and area of the modified rectangle are $2w$, $l+3$, and $A' = 2w(l+3)$, respectively. We are given that $A' = 3A$, so $2w(l+3) = 3wl$. Dividing both sides of this equation by w gives $2(l+3) = 3l$, so $2l+6 = 3l$. Therefore $l = 6$.

12. (MH 11-12 2008) A square has perimeter p and area A. If $A = 2p$, then what is the value of p?

    (a) 54

(b) 48

(c) 36

(d) 32

(e) 24

Solution. Let the side of the square be x. Then its perimeter is $4x$ and its area is x^2, so we are given that $x^2 = 2 \cdot 4x$. So $x^2 = 8x$. Dividing both sides by x, we have $x = 8$. Then $p = 4x = 32$.

Various volume problems

13. (MH 11-12 2008) The volume of a large (solid) cube is 125 cubic inches. A new shape is formed by removing a 1 in $\times$ 1 in $\times$ 1 in cube from one corner of the large cube. The surface area of this new shape in square inches is

(a) 250

(b) 225

(c) 180

(d) 150

(e) 120

Solution. Let us first determine the size of the large cube. If its side is x inches, then its volume is x^3 square inches. So $x^3 = 125$, and we find $x = 5$. Now notice that the surface area does not changed when a small cube is removed from one corner. The surface area of the original cube is $6 \cdot 5^2 = 6 \cdot 25 = 150$ square inches.

14. (MH 9-10 2008) A metal tank in the shape of a right circular cylinder is one-fourth full of water. If 80 mL of water is added, it will be one-third full. What is the volume of the tank?

(a) 960 mL

(b) 320 mL

(c) 240 mL

(d) It can't be determined from teh information given.

Solution. In this problem, the shape of the tank is irrelevant. The key is the difference between one-fourth of the volume and one-third of it. Let the volume of the tank be V. Then we are given that

$$\frac{V}{4} + 80 = \frac{V}{3}$$

$$\frac{V}{3} - \frac{V}{4} = 80$$

$$\frac{4V - 3V}{12} = 80$$

$$\frac{V}{12} = 80$$

$V = 960$ mL

15. (MH 9-10 2008) A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

(a) 10

(b) 25

(c) 36

(d) 50

Solution. Let r and h denote the radius and the height, respectively, of the old jars. (Note that when the diameter is increased by a certain percent, the radius is increased by the same percent.) The old volume is $V = \pi r^2 h$. If we increase the radius by 25%, then the new radius is $\frac{5}{4}r$. Let h' denote the new height, then we want the volume to be the same:

$$\pi \left(\frac{5}{4}r\right)^2 h' = \pi r^2 h$$

$$\left(\frac{5}{4}r\right)^2 h' = r^2 h$$

$$\frac{25}{16} r^2 h' = r^2 h$$

$$\frac{25}{16} h' = h$$

$$h' = \frac{16}{25} h$$

$$h' = \frac{64}{100} h$$

So the height must be decreased by 36%.