Math 111 **Some fundamental properties of logical operations**

Which of the logical operations $\vee$, $\wedge$, $\Rightarrow$, $\Leftrightarrow$ have the properties of commutativity or associativity? Which combinations of the above logical operations and/or $\sim$ have distributivity? That is, which of the following logical equivalences are true? (Note: more combinations could be considered for distributivity, but we will focus on a few most important ones.)

- Commutativity

  $P \vee Q \equiv Q \vee P$

  $P \wedge Q \equiv Q \wedge P$

  $P \Rightarrow Q \equiv Q \Rightarrow P$

  $P \Leftrightarrow Q \equiv Q \Leftrightarrow P$

- Associativity

  $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$

  $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$

  $(P \Rightarrow Q) \Rightarrow R \equiv P \Rightarrow (Q \Rightarrow R)$

  $(P \Leftrightarrow Q) \Leftrightarrow R \equiv P \Leftrightarrow (Q \Leftrightarrow R)$

- Distributivity

  $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

  $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

  $\sim (P \vee Q) \equiv (\sim P) \vee (\sim Q)$

  $\sim (P \wedge Q) \equiv (\sim P) \wedge (\sim Q)$

  $\sim (P \Rightarrow Q) \equiv (\sim P) \Rightarrow (\sim Q)$

  $\sim (P \Leftrightarrow Q) \equiv (\sim P) \Leftrightarrow (\sim Q)$

  $P \Rightarrow (Q \wedge R) \equiv (P \Rightarrow Q) \wedge (P \Rightarrow R)$

  $P \Rightarrow (Q \vee R) \equiv (P \Rightarrow Q) \vee (P \Rightarrow R)$